

Last problem set

Problem 1. Associate to a point z in Poincare disc model point $\frac{2z}{|z|^2+1}$ in Klein's model.

- a) Prove that it is a bijection on points.
- b) Prove that it sends lines to lines.
- c) Prove that it preserves distances.

Problem 2. Prove that circles in Poincare model are Euclidean circles.

Problem 3. Let T be a hyperbolic triangle with angles α, β, γ and lengths of sides a, b, c .

- a) Prove that if $\gamma = \frac{\pi}{2}$ then

$$\operatorname{ch}(c) = \operatorname{ch}(a)\operatorname{ch}(b).$$

- b) Prove that

$$\frac{\operatorname{sh}(a)}{\sin(\alpha)} = \frac{\operatorname{sh}(b)}{\sin(\beta)} = \frac{\operatorname{sh}(c)}{\sin(\gamma)}.$$

- c) Prove that

$$\operatorname{ch}(a) = \operatorname{ch}(b)\operatorname{ch}(c) - \operatorname{sh}(b)\operatorname{sh}(c)\cos(\alpha).$$

Problem 4. a) Find all triangles with angles $\frac{\pi}{p}, \frac{\pi}{q}, \frac{\pi}{r}$, reflecting which one can cover \mathbb{R}^2 . Draw the pictures.

- b) The same question for \mathbb{S}^2 .