Problem set 4

Exercise 1. *Prove that for every two nonintersecting circles S_1 and S_2 there exists an inversion, making them concentric.*

Exercise 2. *Prove that if for a prime p the order of a group G equals to p, then the group is cyclic.*

Exercise 3 (**The Inclusion-Exclusion Principle**)**.** *Consider N objects and some list $P_1, P_2, \ldots, P_n$ of their properties. Let N_i be the number of objects satisfying P_i, N_{ij}, the number of objects satisfying P_i and P_j, and so on. Prove that the number of objects satisfying none of these properties is equal to*

$$N - \sum N_i + \sum_{i_1<i_2} N_{i_1 i_2} - \sum_{i_1<i_2<i_3} N_{i_1 i_2 i_3} + \ldots + (-1)^n N_{123\ldots n}.$$

Problem 1. *Consider four circles such that S_1 and S_3 intersect both S_2 an S_4. Prove that if the points of intersection of S_1 with S_2 and S_3 with S_4 lie on the same circle or line, then the points of intersection of S_1 with S_4 and S_2 with S_3 lie on the same circle or line.*

Problem 2. *Prove that if for a prime p the order of a group G equals to p^2, then the group is abelian.*

Problem 3 (Steiner's Chain)**.** *Suppose that there exists a chain of circles $S_1, S_2, \ldots, S_n$, such that S_i is tangent to S_{i+1} (and S_n is tangent to S_1) and all S_i are tangent to the two fixed circles R_1 and R_2. Prove that there exist infinitely many such chains.*

Problem 4. *Is it possible to draw 9 segments in the plane, so that each of them intersects exactly five other segments?*

Problem 5. *Consider a country with 15 towns. Each of them is connected by a road with 7 others. Prove that it is possible to get from any town to any other.*