Problem set 2

Exercise 1. *Prove that in the multiplication table of a group, every element appears exactly ones in each raw and each column.*

Exercise 2.
1. Let S be the group of isometries preserving a regular 5-gon. Find the order of S, orders of all its elements and write down the corresponding permutations.
2. Prove that a subgroup of a cyclic group is cyclic.
3. Suppose that g is an element of order n in the group G. Prove that for every $k \in \mathbb{N}$ the element g^k has order $\frac{n}{(k,n)}$.

Exercise 3. *Prove that a composition of an odd number of symmetries can not be equal to a composition of an even number of symmetries.*

Problem 1. *Let $ABCD$ be a convex 4-gon and consider four squares constructed on the outside of each of its edges. Prove that the segments connecting the centers of the opposite squares are mutually perpendicular and equal in length.*

Problem 2. *Prove that if we remove two opposite corners from the chessboard, the board cannot be covered by dominoes. (Each domino covers two neighboring cells of the chessboard.)*

Problem 3. *The points $A_1, \ldots, A_n$ form a regular polygon, inscribed in a circle with the center O. A point X lies on the same circle. Prove that the images of the point X under the symmetries with axes $OA_1, OA_2, \ldots, OA_n$ form a regular polygon.*

Problem 4. *Consider n pairs of points on the segment AB, which are symmetric with respect to its center. Half of these points are red, the other are blue. Prove that the sum of distances from the point A to the blue points equals to the sum of distances from the point B to the red points.*