Problem set 12

Problem 1. *Prove that the locus of the intersection points of quadrilaterals $ABCD$ whose sides AB and CD belong to two given lines l_1 and l_2 and sides BC and AD intersect at a given point P is a line passing through the intersection point Q of lines l_1 and l_2.*

Problem 2. *Let O be the intersection point of the diagonals of quadrilateral $ABCD$; let E (resp. F) be the intersection point of the continuations of sides AB and CD (resp. BC and AD). Line EO intersects sides AD and BC at points K and L, respectively, and line FO intersects sides AB and CD at points M and N, respectively. Prove that the intersection point X of lines KN and LM lies on line EF.*

Problem 3. *Points A, B, C lie on line l and points A_1, B_1, C_1 on line l_1. Prove that the intersection points of lines AB_1 and BA_1, BC_1 and CB_1, CA_1 and AC_1 lie on one line*

Problem 4. *Given convex quadrilateral $ABCD$. Let P, Q be the intersection points of the continuations of the opposite sides AB and CD, AD and BC, respectively, R an arbitrary point inside the quadrilateral. Let K, L, M be the intersection point of lines BC and PR, AB and QR, AK and DR, respectively. Prove that points L, M and C lie on one line.*