Problem set 11

Exercise 1. *Prove that if a non-identity affine transformation L sends each point of line l into itself, then all the lines of the form $ML(M)$, where M is an arbitrary point not on l, are parallel to each other.*

Exercise 2. *Prove that if an affine transformation sends a circle into itself, then it is either a rotation or a symmetry.*

Problem 1. *Through every vertex of a triangle two lines are drawn. The lines divide the opposite side of the triangle into three equal parts. Prove that the diagonals connecting opposite vertices of the hexagon formed by these lines intersect at one point.*

Problem 2. *Prove that with the help of a ruler only it is impossible to divide a given segment in halves.*

Problem 3. *A plane is painted three colours. Prove that there are two points of the same colour the distance between which is equal to 1.*

Problem 4. *Given an ellipse E , consider a set of parallel chords of E. Prove that the midpoints of these chords form a diameter of the ellipse (a segment, passing through the center of symmetry) and the tangent lines to E at the endpoints of the diameter are parallel to the chords.*