Problem set 10

Problem 1. *Is it possible to draw 9 segments in the plane, so that each of them intersects exactly five other segments?*

Problem 2. *Consider a country with 15 towns. Each of them is connected by a road with 7 others. Prove that it is possible to get from any town to any other.*

Problem 3. *Nonintersecting diagonals divide a convex $n-$gon into triangles and at each vertex of a polygon an odd number of triangles meet. Prove that n is divisible by 3.*

Problem 4. *Circles $S_1, S_2, \ldots, S_n$ are tangent to two circles R_1 and R_2. Moreover, S_1 is tangent to S_2 at A_1, S_2 is tangent to S_3 at A_2, $\ldots$, S_{n-1} is tangent to S_n at A_{n-1}. Prove that points $A_1, A_2, \ldots, A_n$ lie on the same circle.*

Problem 5. *Denote by $\mu(n)$ the sum of all primitive $n-$th roots of unity.*
a) Compute $\mu(n)$ for $n \leqslant 8$.
b) Prove that if n_1 and n_2 are coprime, then

$$\mu(n_1 n_2) = \mu(n_1)\mu(n_2).$$

b) Compute

$$\sum_{d|n} \mu(d).$$

d) Compute $\mu(n)$.