Problem set 8

Exercise 1. *a) Prove that linear fractions* $\frac{1}{x-\alpha}$ *for* $\alpha \in \mathbb{R}$ *are linearly independent.*
b) Prove that $sin(x)$ *and* $cos(x)$ *are linearly independent.*
c) Prove that for $m \neq n$

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx = 0.$$

d) Prove that functions $1, \cos(x), \cos(2x), \ldots$ *are linearly independent.*

Exercise 2. *a) Prove that* $v_1, v_2, \ldots, v_n$ *are not linearly independent if and only if for some* i

$$v_i \in Span(v_1, \ldots, \widehat{v_i}, \ldots, v_n).$$

b)Prove that intersection of two linear subspaces of V *is again a linear subspace.*

Exercise 3. *Prove that the group* $PGL_2(\mathbb{F}_3)$ *is isomorphic to* $\mathbb{S}_4$.

Exercise 4. *Prove that for every two nonintersecting circles* S_1 *and* S_2 *there exists an inversion, making them concentric.*

Problem 1. *Consider four circles such that* S_1 *and* S_3 *intersect both* S_2 *an* S_4. *Prove that if the points of intersection of* S_1 *with* S_2 *and* S_3 *with* S_4 *lie on the same circle or line, then the points of intersection of* S_1 *with* S_4 *and* S_2 *with* S_3 *lie on the same circle or line.*

Problem 2. *Prove that if for a prime* p *the order of a group* G *equals to* p^2, *then the group is abelian.*

Problem 3 (Steiner's Chain)**.** *Suppose that there exists a chain of circles* $S_1, S_2, \ldots, S_n$, *such that* S_i *is tangent to* S_{i+1} *(and* S_n *is tangent to* S_1*) and all* S_i *are tangent to the two fixed circles* R_1 *and* R_2. *Prove that there exist infinitely many such chains.*

Problem 4. *Let* S *be a set of* $n+1$ *integers from* 1 *to* $2n$. *Prove that at least two elements in* S *are coprime.*