Problem set 4

Exercise 1. *Prove that in the multiplication table of a group, every element appears exactly ones in each raw and each column.*

Exercise 2.
1. Let S be the group of isometries preserving a regular 5-gon. Find the order of S, orders of all its elements and write down the corresponding permutations.
2. Prove that a subgroup of a cyclic group is cyclic.
3. Suppose that g is an element of order n in the group G. Prove that for every $k \in \mathbb{N}$ the element g^k has order $\frac{n}{(k,n)}$.

Exercise 3. *Prove that isomorphism is an equivalence relation.*

Exercise 4. *Prove that the group of orientation preserving isometries of $\mathbb{R}^3$ preserving a cube is isomorphic to $\mathbb{S}_4$.*

Problem 1. *Consider a sequence $\{a_1, a_2, \ldots, a_n\}$ of integer numbers with the sum equal to 1. Prove that for exactly one of its cyclic shifts*

$$\{a_1, a_2, \ldots, a_n\}, \{a_2, a_3, \ldots, a_1\}, \{a_n, a_1, \ldots, a_{n-1}\}$$

all the partial sums are positive. Deduce from this the formula for Catalan numbers.

Problem 2. 44 *owls are sitting on* 44 *trees planted in a circle (each tree is occupied by one owl). From time to time two of the owls move to the adjacent trees in the opposite directions: one of them moves clockwise and the other counterclockwise. Is it possible that at some moment all the owls end up on the same tree?*