Problem set 2

Exercise 1.
1. Prove that the following identities hold in any group G :

$$(g_1 \cdot g_2)^{-1} = g_2^{-1} \cdot g_1^{-1},$$
$$(g_1 \cdot g_2 \cdot \ldots \cdot g_n)^{-1} = g_n^{-1} \cdot \ldots \cdot g_2^{-1} \cdot g_1^{-1}.$$

2. Write down multiplication table for the group of isometries preserving a square.
3. Write down multiplication table for the group of isometries preserving a rhombus which is not a square.
4. Prove that if in the group G the square of every element is the identity, then G is abelian.

Exercise 2. *Prove that a bounded figure in $\mathbb{R}^2$ cannot have more than one center of symmetry.*

Exercise 3. *Prove that*

$$\sum_{k=q}^{n} \binom{n}{k}\binom{k}{q} = 2^{n-q}\binom{n}{q}$$

Exercise 4. *Find a number of ways to cut an $n \times 1$ strip into 2×1 and 1×1 tiles.*

Exercise 5. *Prove that a composition of an odd number of symmetries can not be equal to a composition of an even number of symmetries.*

Problem 1. *Let $ABCD$ be a convex 4-gon and consider four squares constructed on the outside of each of its edges. Prove that the segments connecting the centers of the opposite squares are mutually perpendicular and equal in length.*

Problem 2 (Lagrange theorem). *Let G be a group of finite order. Prove that for every element g*

$$g^{|G|} = 1.$$

Problem 3 (I). *Prove that if we remove two opposite corners from the chessboard, the board cannot be covered by dominoes. (Each domino covers two neighboring cells of the chessboard.)*

Problem 4 (Erdős-Szekeres theorem). *Prove that for any $n, m \in \mathbb{N}$, every sequence of $nm + 1$ distinct real numbers contains an increasing subsequence of length $n + 1$ or a decreasing subsequence of length $m + 1$.*